STOCHASTIC GRADIENT DESCENT APPROACH WITH STANDARD ERROR AND ITS APPLICATION TO FINANCIAL PORTFOLIO OPTIMIZATION PROBLEMS

STEPHANIE SU SEE WENG

A thesis submitted in
fulfilment of the requirement for the award of the
Doctor of Philosophy in Science

Faculty of Applied Sciences and Technology
Universiti Tun Hussein Onn Malaysia

JUNE 2023

ACKNOWLEDGMENT

My sincere appreciation goes to my supervisor Dr. Kek Sie Long. His contribution and constructive criticism have pushed me to expend the kind of effort I have exerted to make this work as perfect as possible. Thanks to him, I have experienced actual research, and my knowledge of the subject matter has been broadened.

My utmost regard also goes to my parents, who painstakingly laid the foundation for my education, giving it all it takes. I am also grateful to my loving husband, who has endured so much stress and discomfort just for me. Last but not least, appreciation also goes to everyone involved directly or indirectly in the compilation of this thesis.

ABSTRACT

Stochastic optimization in financial portfolio investment is a challenging task. In this thesis, a computational approach is proposed to solve the financial portfolio optimization problems. For this purpose, the stochastic gradient descent (SGD) method is overviewed, and its recent variant, the adaptive moment estimation (Adam) approach, is investigated. Notice that the updating rule in the Adam algorithm consists of the component of the second moment of past gradients, which is also known as the standard deviation of gradients. Hence, in our study, the computational algorithm mainly focuses on the SGD and Adam algorithms, and the standard error (SE) of sampling of the past gradients is added to the updating rule. So, the convergence rate can be fastened with fewer iteration numbers. On this basis, the proposed algorithm is known as the AdamSE algorithm. On the other hand, the application of the SGD, Adam and AdamSE algorithms to financial portfolio optimization models for the Employees Provident Fund (EPF) is examined. Here, a simulated mean-variance model is defined by using the parameters of the expected return and the covariance matrix from the classical mean-variance model, and the performance of algorithms is observed. Then, a mean-value at risk (mean-VaR) model is introduced, and the standard error of sampling of past gradients is associated with the AdamSE algorithm for obtaining different iteration steps toward the optimal solution. Next, a Black-Litterman model is studied, and different types of gradients in the measure of the central tendency of mean, median and mode gradients are employed in the AdamSE algorithm to express the efficiency of the algorithm. Accordingly, through these financial portfolio optimization models, the features of the AdamSE algorithm are demonstrated. Therefore, the efficiency of the proposed algorithm is proven. In conclusion, the practical application of these SGD algorithms to financial portfolio optimization problems is verified.

ABSTRAK

Pengoptimuman berstokastik dalam pelaburan portfolio kewangan adalah tugas yang mencabar. Dalam tesis ini, satu kaedah pengiraan dicadangkan untuk menyelesaikan masalah pengoptimuman portfolio kewangan. Untuk tujuan ini, kaedah penurunan kecerunan stokastik (SGD) ditinjau secara keseluruhan, dan varian terbarunya, kaedah anggaran momen penyesuaian (Adam), dikaji. Diperhatikan bahawa peraturan pengemaskinian dalam algoritma Adam terdiri daripada komponen momen kedua kecerunan lalu, yang juga dikenali sebagai sisihan piawai kecerunan. Oleh itu, dalam kajian ini, algoritma pengiraan tertumpu terutamanya pada algoritma SGD dan Adam, dan ralat piawai (SE) pensampelan kecerunan lalu ditambahkan pada peraturan pengemaskinian. Jadi, kadar penumpuan boleh dipertingkatkan dengan nombor lelaran yang lebih sedikit. Berasakan ini, algoritma yang dicadangkan dikenali sebagai algoritma AdamSE. Sebaliknya, penggunaan algoritma SGD, Adam dan AdamSE terhadap model pengoptimuman portfolio kewangan untuk Kumpulan Wang Simpanan Pekerja (KWSP) diperiksa. Di sini, model min-varians simulasi ditakrifkan dengan menggunakan parameter pulangan yang dijangkakan dan matriks kovarians daripada model min-varians klasik, dan prestasi algoritma diperhatikan. Kemudian, model nilai min berisiko (min-VaR) diperkenalkan, dan ralat piawai pensampelan kecerunan lalu disatukan dengan algoritma AdamSE untuk mendapatkan langkah lelaran yang berbeza ke arah penyelesaian optimum. Seterusnya, model *Black-Litterman* dikaji, dan pelbagai jenis kecerunan dalam ukuran kecenderungan memusat, iaitu min, median dan mod kecerunan digunakan dalam algoritma AdamSE untuk menyatakan kecekapan algoritma. Sehubungan itu, melalui model pengoptimuman portfolio kewangan ini, ciri-ciri algoritma AdamSE ditunjukkan. Dengan itu, kecekapan algoritma yang dicadangkan terbukti. Kesimpulannya, penggunaan praktikal algoritma SGD ini untuk masalah pengoptimuman portfolio kewangan disahkan.

CONTENTS

	TITLE		**i**
	DECLARATION		**ii**
	ACKNOWLEDGEMENT		**iii**
	ABSTRACT		**iv**
	ABSTRAK		**v**
	CONTENTS		**vi**
	LIST OF TABLES		**ix**
	LIST OF FIGURES		**xi**
	LIST OF SYMBOLS AND ABBREVIATION		**xiii**
CHAPTER 1	**INTRODUCTION**		**1**
	1.1	Research background	1
	1.2	Problem statement	3
	1.3	Research objectives	4
	1.4	Scope of study	5
	1.5	Significance of study	5
	1.6	Structure of thesis	7
	1.7	Chapter summary	8
CHAPTER 2	**LITERATURE REVIEW**		**9**
	2.1	Introduction to gradient descent methods	9
	2.2	Development of stochastic gradient descent	10
	2.3	Adaptive moment estimation	10
	2.4	Portfolio	11
	2.5	Risk	11
	2.6	Return	12
	2.7	Relation between risk and return	12
	2.8	Diversification	13

	2.9	Portfolio optimization models	14
	2.9.1	Mean-variance model	14
	2.9.2	Mean-value at risk model	15
	2.9.3	Black-Litterman model	15
	2.10	Recent studies on stochastic gradient methods to portfolio optimization	17
	2.11	Employees Provident Fund: an overview	17
	2.12	Chapter summary	18
CHAPTER 3	**METHODOLOGY**		**19**
	3.1	Introduction	19
	3.2	Research framework	20
	3.3	General optimization model	21
	3.4	Necessary conditions	22
	3.5	Stochastic gradient descent algorithm	22
	3.6	Adaptive moment estimation algorithm	23
	3.7	Adaptive moment estimation algorithm with standard error	26
	3.8	Data collection	29
	3.9	Chapter summary	36
CHAPTER 4	**MEAN-VARIANCE PORTFOLIO OPTIMIZATION MODEL FOR EMPLOYEES PROVIDENT FUND**		**37**
	4.1	Introduction to mean-variance model	37
	4.2	Parameters in portfolio model	38
	4.3	Lagrange function and necessary conditions	41
	4.4	Analytical solution for deterministic case	42
	4.5	Simulation of mean and covariance	43
	4.6	Optimal weight allocation	44
	4.7	Performance of algorithms	46
	4.8	Efficient frontier	50
	4.9	Portfolio Risk	52
	4.10	Chapter summary	56

CHAPTER 5	**MEAN-VAR PORTFOLIO OPTIMIZATION MODEL FOR EMPLOYEES PROVIDENT FUND**	**57**
	5.1 Introduction to mean-value at risk model	57
	5.2 Derivation of necessary conditions	60
	5.3 Analytical solution of deterministic model	60
	5.4 Results and discussion	62
	5.5 Chapter summary	65
CHAPTER 6	**BLACK-LITTERMAN PORTFOLIO OPTIMIZATION MODEL FOR EMPOYEES PROVIDENT FUND**	**66**
	6.1 Introduction to Black-Litterman Model	66
	6.2 Expected return	67
	6.2.1 Implied return	67
	6.2.2 Required return	68
	6.2.3 Combined return	69
	6.3 Investors' view	70
	6.4 Model formulation	73
	6.5 Alternative gradients	75
	6.6 Optimal weight allocation	76
	6.7 Convergence results	77
	6.8 Portfolio risk	78
	6.9 Chapter summary	79
CHAPTER 7	**CONCLUSION AND RECOMMENDATION**	**80**
	7.1 Contribution of study	80
	7.2 Limitation of study	81
	7.3 Future direction and recommendation	82
	7.4 Chapter summary	82
	REFERENCES	**83**
	VITA	**92**

LIST OF TABLES

1.1	Ten assets from EPF	5
3.1	The ten selected stocks	31
4.1	Optimal portfolio weights for classical mean-variance model	44
4.2	Optimal portfolio weights for simulated mean-variance model	44
4.3	Capital allocation with optimal weight from classical mean-variance model	45
4.4	Capital allocation with optimal weight from simulated mean-variance model	46
4.5	Performance of algorithms for mean-variance models	46
5.1	Optimal portfolio weight for the mean-VaR model	62
5.2	Number of iterations for algorithms	63
5.3	Portfolio VaR and risk for three different algorithms	63
5.4	Optimal portfolio weights for mean-VaR model with different sample sizes in AdamSE algorithm	63
5.5	Portfolio VaR and risk for different numbers of samples	64
5.6	Performance of AdamSE algorithm for mean-VaR model	64
5.7	Portfolio risk for mean-VaR model with different confidence levels	65
5.8	Portfolio risk for classical mean-variance model with different confidence levels	65
6.1	Current and target stock price of the selected ten stocks	71
6.2	Optimal portfolio weight for Black-Litterman model	76
6.3	Number of iterations for the different algorithms	76
6.4	Optimal portfolio weights for Black-Litterman model with different types of gradients in AdamSE algorithm	77

6.5 Convergence performance of AdamSE algorithm with different types of gradients 77
6.6 Portfolio risk for Black-Litterman model with different types of gradients in AdamSE algorithm 78

LIST OF FIGURES

3.1	Flowchart of research framework	20
3.2	Stock price change for IJM Corporation Berhad	31
3.3	Stock price change for Bermaz Auto Berhad	32
3.4	Stock price change for Yinson Holdings Berhad	32
3.5	Stock price change for Malaysia Building Society Berhad	33
3.6	Stock price change for Kuala Lumpur Kepong Berhad	33
3.7	Stock price change for Malaysian Resources Corporation Berhad	34
3.8	Stock price change for Globetronics Technology Berhad	34
3.9	Stock price change for Axiata Group Berhad	35
3.10	Stock price change for Malaysia Airports Holdings Berhad	35
3.11	Stock price change for Tenaga Nasional Berhad	36
4.1	Pseudo-code for simulation of mean and covariance of portfolio	43
4.2	Convergence behavior for SGD algorithm, classical mean-variance model	47
4.3	Convergence behavior for Adam algorithm, classical mean-variance model	48
4.4	Convergence behavior for AdamSE algorithm, classical mean-variance model	48
4.5	Convergence behavior for SGD algorithm, simulated mean-variance model	49
4.6	Convergence behavior for Adam algorithm, simulated mean-variance model	49
4.7	Convergence behavior for AdamSE algorithm, simulated mean-variance model	50

4.8 Efficient frontier for portfolio in classical mean-variance model 51

4.9 Efficient frontier for portfolio in simulated mean-variance model 51

4.10 Portfolio risk from SGD algorithm, classical mean-variance portfolio model 53

4.11 Portfolio risk from Adam algorithm, classical mean-variance portfolio model 53

4. 12 Portfolio risk from AdamSE algorithm, classical mean-variance portfolio model 53

4.13 Portfolio risk from SGD algorithm, simulated mean-variance portfolio model 53

4.14 Portfolio risk from Adam algorithm, simulated mean-variance portfolio model 53

4.15 Portfolio risk from AdamSE algorithm, simulated mean-variance portfolio model 53

LIST OF SYMBOLS AND ABBREVIATIONS

E	-	Expectation operator
g	-	Gradient
I	-	Total return
$\mathbf{I}$	-	Vector with 1s elements
j	-	Random index
k	-	Number of iterations
L	-	Lagrange function
m	-	Exponential decaying averages of past gradients
n	-	Number of samples
$\mathbf{P}$	-	Matrix of the investor's view
$\mathbf{q}$	-	Vector of the investor's view
r	-	Rate of return
$\overline{r}$	-	Arithmetic mean of the rate of return
R	-	Target return
$\hat{s}$	-	Standard error of the bias-corrected first-moment estimate
v	-	Past squared gradients
$\mathbf{w}$	-	Vector of the weight
α	-	Step size/learning rate
δ	-	Smoothing term
ε	-	Tolerance
λ	-	Lagrange multiplier
$\boldsymbol{\mu}$	-	Portfolios return mean
$\overline{\boldsymbol{\mu}}$	-	Vector of expected return
π	-	Geometric mean of the rate of return
$\boldsymbol{\pi}$	-	Vector of the market rate of return
σ	-	Population standard deviation

τ	-	Scaling factor
Δ	-	Effective step size
∇	-	Gradient descent
$\mathbf{\Omega}$	-	Diagonal covariance matrix
$\mathbf{\Sigma}$	-	Covariance matrix
$\mathbf{x}^{(0)}$	-	Initial value
β_1	-	Exponential decay rate for the first-moment estimates
β_2	-	Exponential decay rate for the second-moment estimates
Adadelta	-	Adaptive delta
Adagrad	-	Adaptive gradient
Adam	-	Adaptive moment estimation
AdamSE	-	Adam with standard error
CAPM	-	Capital Asset Pricing Model
EPF	-	Employees' Provident Fund
NAG	-	Nesterov acceleration gradient
RMSprop	-	Root mean square propagation
SE	-	Standard error
SGD	-	Stochastic gradient descent

CHAPTER 1

INTRODUCTION

This chapter gives a brief research background to the study. The problem statement of the study is described, and the research objectives are established. In addition, the scope and significance of the study are mentioned. Later, the structure of the thesis is outlined, and a summary of the chapter is given.

1.1 Research background

Finance is generally defined as the art and science of managing wealth (Paramasivan and Subramanian, 2009). The impact of finance ranges from an individual to a community and from a country to other countries worldwide. Therefore, finance constantly interacts with economics in the daily activities involving money usage. Furthermore, financial portfolio optimization is one of the most common financial investment problems encountered by financial practitioners (Bailey and Prado, 2013). Here, a financial portfolio refers to a set of financial assets (Rutkauskas and Stankevičiene, 2003), for example, cash, stocks, bonds, mutual funds, and bank deposits. Thus, financial portfolio optimization aims to make the portfolio superior to other portfolios based on several criteria, such as minimum risk and maximum return (Chin, Chendra and Sukmana, 2018).

In the 1950s, Harry Markowitz developed the first mathematical diversification model for portfolio optimization. Since then, the mathematical model has been known as the Markowitz model in financial investment. In the Markowitz model, the portfolio's return is given by the portfolio's expected return and the variance of its return measures the risk of the portfolio. On this basis, the Markowitz model is also called the mean-variance model. Nonetheless, Gencay and Selcuk (2004) argued that variance is not a good tool to measure the risk of a portfolio. They proposed a new measurement method for a portfolio's risk, which is popularly known as the value at risk (VaR). Hence, variance in the mean-variance model can be replaced by VaR to form the mean-VaR model.

In addition, stock prices are influenced by the company's action plans and policies, where rising and dropping stock prices are uncertain and cannot be perfectly beaten. Therefore, estimating stock prices using historical stock price data is less accurate in predicting stock prices in the future. Investors' perceptions of current stock performance are also crucial in predicting stock prices. Therefore, another portfolio optimization model, namely the Black-Litterman model (Black and Litterman, 1991), was established to combine investor views toward the trend of stock prices with historical data on stock prices.

On the other hand, in solving stochastic optimization problems (Ge *et al.*, 2015), the stochastic gradient descent (SGD) method is the fundamental optimization algorithm among stochastic optimization techniques. The SGD method is an imprecise but useful stochastic optimization algorithm (Chollet, 2021), and many variants of the SGD method have been developed. Recently, the adaptive moment estimation (Adam) approach is one of the most popular iterative algorithms (Sun, 2020) among the SGD variants. Practically, the effectiveness of optimization algorithm is determined by two metrics. The first metric is the convergence speed that presents the iteration process of reaching the global optimum. The second metric is the generalization that shows the algorithm performance in dealing with new data.

Since the nature of portfolio risk involves randomness and uncertainty, financial portfolio optimization models shall be solved using the SGD method for a better outcome. Hence, our main aim in this thesis is twofold. The first is to improve the Adam algorithm, especially the convergence speed, and the second is to apply the SGD method, Adam and the improved algorithm to financial portfolio optimization models. So, the application of the SGD methods in handling financial portfolio optimization problems can be explored and verified. Specifically, the financial portfolio optimization model for the Employees Provident Fund (EPF) is constructed for demonstration in the study.

1.2 Problem statement

In a financial portfolio optimization problem, mean and variance are two essential elements measured from past historical stock prices of a portfolio. The portfolio expected return presents the reward desired to be received from the portfolio investment. While, the variance reveals the risk of the portfolio, which is the deviation of the return from the expected level. As the stock market is uncertain and random, the stock market will not be perfectly beaten, and the movement of stock prices cannot be accurately predicted in the following period. Thus, the volatility of stock prices can cause investors to receive losses. Although many investors are risk inversion oriented whose prefer lower returns with known risks and understand that higher risk gives a higher return, they look for low risk and high reward from financial portfolio investment. So, the issue of losses in financial portfolio investment always happens.

The usefulness of the mean-variance model can provide some insights to investors, for example, to minimize the portfolio risk, maximize the portfolio return, and allocate portfolios optimally. The stock markets are efficient, and investors can only access the available information, such as the expected return, variance, and covariance of securities. With the formulation of the mean-variance model, investors minimize the risk of portfolios at a level of return given. On this basis, an optimal portfolio is determined upon the optimal weights that are resulted from solving the mean-variance model. However, the calculation of the mean and variance based on the past historical stock prices that are applied in the financial portfolio model does not indicate the current movement of the stock prices. Hence, using deterministic optimization techniques to solve financial portfolio problems only gives an ideal optimal solution. These deterministic techniques cannot provide a satisfactory solution that reflects the uncertainty and randomness in financial portfolio optimization.

In addition, stochastic optimization of financial portfolio problems is a realistic resolution, considering the mean and variance of the portfolio in dealing with uncertainty and randomness in the financial portfolio model. Some stochastic approaches, like stochastic programming, genetic algorithm, and random walk, are commonly used for solving financial portfolio optimization problems (Cornuejols and Tutuncu, 2006; Rani, 2012). These approaches give the optimal solution when the optimality conditions are satisfied. However, the computational process is complex, and some calculation stages, which are stochastic gradient and solution updating stages, must be considered before reaching the optimal solution to give the investment decision.

Due to the rapid development of SGD methods, it has been argued that SGD methods might provide more accurate solutions in financial portfolio optimization than other existing stochastic optimization methods. Although the SGD and Adam algorithms are effective in dealing with machine learning problems, the slow convergence of the iterative process due to the existence of random disturbances is the weakness of the algorithms. According to (Keskar and Socher, 2017), the learning rate of the Adam approach is low at the convergence stage, which affects the effectiveness of convergence and slows down the solution process. Thus, this drawback shall be further improved.

Therefore, the motivation of this thesis is to propose an efficient computational algorithm under a stochastic environment for solving financial portfolio optimization problems. The sampling error from the sampling theory is employed in the solution method process for convergence. Past historical stock prices for the top 30 equity holdings in the EPF are used to construct the portfolio optimization models. We expect to provide the optimal decision for financial portfolio investment problems for the EPF at the end of our study.

1.3 Research objectives

The essential variables of financial portfolio optimization model are the expected return and variance of the expected return, which is also known as the risk. Since portfolio risk is uncertain and randomly changing, portfolio uncertainty should be considered in risk estimation. Moreover, the effectiveness of the SGD method in solving stochastic optimization problems has been well-defined as a practical method in engineering and sciences. Therefore, our primary goal is to propose an efficient computational algorithm with better convergence by improving the SGD and Adam algorithms. Later, the application of this computational method to financial portfolio optimization problems is further explored. With this, the following research objectives are established.

(a) To propose an efficient computational algorithm based on the stochastic gradient for solving financial portfolio optimization problems.

(b) To improve the updating rule in the Adam algorithm using the standard error from the sampling theory for accelerating the convergence rate and reducing iteration numbers.

(c) To verify the application of the algorithms of SGD, Adam and the proposed algorithm for solving financial portfolio optimization problems through algorithm performance comparison.

(d) To demonstrate the features of the proposed algorithm with different sample sizes and types of past gradients in terms of mean, median and mode gradients.

1.4 Scope of study

This study covers the application of the algorithms of SGD, Adam, and the proposed algorithm to financial portfolio optimization problems. The proposed algorithm is equipped with the standard error, one of the sampling theory errors. So, this standard error will improve the Adam algorithm, and the proposed algorithm is known as the Adam with standard error (AdamSE) algorithm. In our study, three financial portfolio optimization models, namely the mean-variance model, the mean-VaR model, and the Black-Litterman model, for the EPF, are constructed. Hence, ten assets from the top 30 equity holdings of the EPF were selected on 31 March 2020, as shown in Table 1.1. The weekly historical data of stock prices for these ten assets, which are from 4 January 2015 to 29 December 2019, and available at Investing.com, was chosen.

Table 1.1: Ten assets from EPF

No.	Assets from EPF
1	IJM Corporation Berhad
2	Bermaz Auto Berhad
3	Yinson Holdings Berhad
4	Malaysia Building Society Berhad
5	Kuala Lumpur Kepong Berhad
6	Malaysian Resources Corporation Berhad
7	Globetronics Technology Berhad
8	Axiata Group Berhad
9	Malaysia Airports Holdings Berhad
10	Tenaga Nasional Bhd.

1.5 Significance of study

Stochastic optimization methods with the stochastic gradient provide practical applications in engineering and sciences. The rapid development of computational algorithms brings the SGD method to actively solve optimization problems in random and uncertain environments. In applying the SGD and Adam algorithms, the algorithm convergence will be mainly concerned so that better algorithm efficiency is guaranteed. For this reason, in our study, an efficient computational algorithm is proposed by adding the standard error of sampling theory to improve the updating rule of the Adam algorithm, in turn accelerating the convergence rate with a reduction of iteration steps. This computational algorithm is known as the AdamSE algorithm. Moreover, the application of SGD, Adam and AdamSE algorithms to financial portfolio optimization problems is illustrated.

Therefore, in our study, some significant contributions are claimed as follows,

(a) Applying the standard error of sampling to improve the convergence rate of the Adam algorithm. For doing this, the sample of past gradients is considered, and the standard error is calculated. After that, the standard error replaces the standard deviation in the Adam algorithm to be the component in the updating rule. With this, the number of iterations of the AdamSE algorithm is significantly reduced.

(b) Generating some samples of past gradients to observe the performance of the AdamSE algorithm. Through sampling, the standard error for some samples is calculated, and the convergence rate of the AdamSE algorithm is recorded. Hence, the iterative algorithm associated with sampling gives significant work to the stochastic optimization community.

(c) Testing the AdamSE algorithm with different types of gradients in terms of mean, median and mode of past gradients. This work presents the usefulness of the measures of the central tendency in the development of the SGD algorithm. Using these types of gradients in the AdamSE algorithm confirms brings new insight into stochastic optimization.

(d) Constructing the financial portfolio optimization models, namely the mean-variance model, the mean-VaR model and the Black-Litterman model, for the EPF. The optimal weights of these models are determined by using the SGD, Adam and AdmSE algorithms. Thus, the application of these algorithms to financial portfolio optimization problems is verified.

1.6 Structure of thesis

The content of this thesis is outlined as follows. In Chapter 1, the introduction of the study is given, where the background of the study is stated. The problem statement describes the problem to be resolved in the study, and the objectives of the study are then established for handling the problem. In addition, the scope of the study is mentioned, and the significance of the study is highlighted. Follow from this, the structure of the thesis is presented, and a chapter summary is provided.

In Chapter 2, an introduction to the gradient descent approach is given, and the development of the SGD method, including its recent variant, namely the Adam algorithm, is presented. Then, the basic terminology of the financial portfolio is provided. The financial portfolio optimization models, which are the mean-variance model, the mean-VaR model and the Black-Litterman model, are reviewed.

In Chapter 3, the research framework is provided, and the general optimization problem is introduced. The Lagrange function is defined and the first-order necessary conditions are derived. The calculation procedure of the SGD, Adam and AdamSE algorithms is provided. Besides, the data collection of the historical stock prices of the ten stocks for the EPF is displayed in a graphical form.

In Chapter 4, the mean-variance model for the EPF is studied. The parameters of the expected return and covariance matrix are calculated for the classical mean-variance model. For the purpose of a more reliable solution, these parameters are employed to simulate new parameters of the expected mean and covariance matrix, and another model, which is called the simulated mean-variance model, is defined. Using the SGD, Adam and AdamSE algorithms, the optimal weights and the portfolio risk are obtained. The efficient frontier of the portfolio is expressed and the algorithm performance is discussed.

In Chapter 5, the mean-VaR model for the EPF is studied. The concept of the VaR is explained and the mean-VaR model is defined. Then, the optimal weights and the portfolio risk are obtained by using the SGD, Adam and AdamSE algorithms. To highlight the feature of the AdamSE algorithm, some samples of the past gradients are considered and the corresponding standard error is computed. The performance of the algorithm is observed and discussed.

In Chapter 6, the Black-Litterman model for the EPF is studied. Unlike the classical mean-variance model, the investors' views of the portfolio are taken into consideration in the Black-Litterman model. The expected return used in the model incorporates the market return and required return into the model formulation. Using the SGD, Adam and AdamSE algorithms, the optimal weights and the portfolio risk are determined. To demonstrate the feature of the AdamSE algorithm, different types of gradients in terms of mean, median and mode gradients are considered in the algorithm. The performance of the algorithm is compared and discussed.

In Chapter 7, a conclusion of the study is delivered. The contributions of the study are significantly reported, and the achievement of the objectives of the study is mentioned. The limitation of the study is pointed out and some recommendations for future research are suggested.

1.7 Chapter summary

This chapter provided the background of the study, and the problem statement of the study. Follow from this, the objectives of the study were established, the scope of the study was mentioned, and the significance of the study was expressed. At the end, the structure of the thesis was outlined. In the next chapter, a review of the methods used and the financial portfolio optimization problems will be conducted.

CHAPTER 2

LITERATURE REVIEW

In this chapter, an introduction to gradient descent methods is delivered, where the development of the stochastic gradient descent (SGD) method and its recent variant, namely the adaptive moment estimation (Adam) approach, are covered. Also, this chapter introduces the concept and basic terminology of financial portfolio optimization. Moreover, financial portfolio optimization models, including the mean-variance model, mean-VaR model, and Black-Litterman model, are reviewed. Finally, a summary of this chapter is given.

2.1 Introduction to gradient descent methods

The gradient descent method is a first-order optimization algorithm that only considers the first derivative when updating parameters during iteration. Here, the iteration is referred to as the complete process of repeatedly computing gradients and updating points. The algorithm calculates the gradient of the objective function at the current point and then updates parameters in the opposite direction of the gradient during iteration until local minima are reached (Netrapalli, 2019). There are three variants of the gradient descent method, which are batch gradient descent, mini-batch gradient descent, and stochastic gradient descent. These methods differ in the amount of data used to calculate the gradient of the objective function. Often, they have a trade-off between the accuracy of parameter updates and the time required to perform the updates (Ruder, 2016).

2.2 Development of stochastic gradient descent

Stochastic gradient descent (SGD) is preceded by stochastic approximation (SA), which was first described by Robbins and Monro (1951). Then Kiefer and Wolfowitz (1952) further explored this approach for a regression function. It is a relatively small leap from the Robbins and Monro method to the Kiefer Wolfowitz method, which only reframes the problem to reach the SGD approach. This literature statement is widely cited as the predecessor of the SGD approach.

The SGD method drastically simplifies the classical gradient method, iterating only one or a small batch of randomly selected samples rather than the entire dataset (Andrearczyk, 2017; Roset, 2019). As a result, this approach usually performs the computation tasks very quickly. Furthermore, because the SGD method does not need to remember examples accessed in previous iterations, it can process the examples dynamically in a deployed system (Bottou, 2012). However, due to the frequent update of the SGD method, the variance of the objective function is large, and resulting in a large fluctuation of the objective function. The fluctuation keeps the SGD method overshooting but may make it jump to a new and possibly better local minimum (Ishibashi, 2017).

2.3 Adaptive moment estimation

The adaptive moment estimation (Adam) algorithm (Kingma and Ba, 2015) is a combination of the root mean square propagation (RMSprop) algorithm (Muhamedyev, 2015) and the SGD with momentum (Polyak, 1964). It uses the squared gradients to scale the learning rate, as done in the RMSprop algorithm. Also, it takes advantage of momentum using the moving average of the gradient instead of the gradient itself, like the SGD with momentum. Although the Adam algorithm is one of the best optimizers compared with other SGD algorithms, it is not perfect (Liu *et al.*, 2023). It may not converge to the optimal solution of objective functions, and the weight decay issue may occur. Its advantages include computational efficiency, memory efficiency, working well on large data sets, and handling sparse gradients on noisy datasets.

In literature, the Adam algorithm is one of the most popular variants of the SGD approach (Sun, 2020). The convergence speed of the Adam algorithm is faster than the SGD approach. However, the final convergence result is not as good as the

SGD approach. A study in (Keskar and Socher, 2017) found that the Adam learning rate was too low in the later stage, affecting the effectiveness of convergence. Therefore, an improvement in the convergence rate of the Adam algorithm should be addressed. Furthermore, the application of the SGD approach and Adam algorithm to financial portfolio optimization problems has to be investigated.

2.4 Portfolio

From a financial perspective, a portfolio is referred to a group of investment assets held by professional institutions, investors or individuals (Zhang, 2011; Mariak and Mitkova, 2016). A financial portfolio typically consists of several financial asset classes, such as stocks, equities, mutual funds, bonds, and cash equivalents (Gupta *et al*., 2020). Modern portfolio theory (MPT) is the most groundbreaking portfolio concept, which improves classical investment models. Unlike classical security analysis, MPT shifts its focus from analyzing the characteristics of individual investments to determining the statistical relationships between individual securities that make up the overall portfolio (Elton and Gruber, 1997). The goal of MPT is to select a portfolio of assets whose collective risk is lower than that of any individual asset with the concept of diversification at a given expected return (Elton *et al*., 2014).

2.5 Risk

Risk is an important term in portfolio optimization models, and it is referred to the uncertainty of an outcome of actions and events, whether positive opportunities or harmful threats (Gupta, Sharma and Trivedi, 2016). Risk and uncertainty are often seen as synonyms but there is a different meaning to understand. Risk is a situation in which the possibility of a possible outcome can be quantified and measured, while uncertainty is a situation in which the possibility cannot be measured (Gough, 1988). In mathematical terms, a risk is the product of the probability measure of an unwanted event and its consequences (Boholm, 2019). In the financial field, a risk is defined as the variability of equity owners' net return due to external financing instruments (Fogarasi et al., 2015).

REFERENCES

Abad, P. and Benito, S. (2005). *Using The Nelson and Siegel Model of The term Structure in Value at Risk Estimation*. Universidad de Barcelona: Working Paper.

Abdul Razak, H. N., Maasar, Mohd. A., Hafidzuddin, N. H. and Lee, E. S. C. (2019). Portfolio optimization of risky assets using mean-variance and mean-CVaR. *Journal of Academia*, 7(1), 25–32.

Abdul Samad, M. F. (1996). *Performance of Group-Affiliated Firms: A Study of Politically-Affiliated Business Group in Malaysia.* University of Rhode Island: Ph.D. Thesis.

Alexander, G. J. and Baptista, A. M. (2002). Economic implications of using a mean-VaR model for portfolio selection: A comparison with mean-variance analysis. *Journal of Economic Dynamics and Control*, 26 (7-8), 1159–1193.

Almisher, M. A. and Kish, R. J. (2000). Accounting betas-An ex anti proxy for risk within the IPO market. *Journal of Financial and Strategic Decisions*, 13, 23–34.

Andrearczyk, V. (2017). *Deep learning for texture and dynamic texture analysis.* Dublin City University: Ph.D. Thesis.

Bailey, D. H. and Prado, M. L. de. (2013). An open-source implementation of the critical-line algorithm for portfolio optimization. *Algorithms*, 6(1), 169–196.

Becker, F. and Gürtler, M. (2008). *Quantitative Forecast Model for the Application of the Black-Litterman Approach*. University of Braunschweig: Working Paper Series No. IF27V2.

Bertsimas, D., Gupta, V. and Paschalidis, I. C. (2012). Inverse Optimization: A New Perspective on the Black-Litterman Model. *Operations Research*, 60(6), 1389-1403.

Bevan, A. and Winkelmann, K. (1998). *Using the Black-Litterman Global Asset Allocation Model: Three Years of Practical Experience.* United Kingdom: Goldman, Sachs and Co.

Black, F. and Litterman, R. (1991). Asset Allocation: Combining Investor Views with Market Equilibrium. *The Journal of Fixed Income*, 1(2), 7-18.

Black, F. and Litterman, R. (1991). *Global Asset Allocation With Equities, Bonds, and Currencies*. United Kingdom: Goldman, Sachs and Co.

Black, F. and Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*, 48(5), 28-43.

Bodnar, T., Ivasiuk, D., Parolya, N. and Schmid, W. (2020). Mean-variance efficiency of optimal power and logarithmic utility portfolios. *Mathematics and Financial Economics*, 14(3), 675-698.

Boholm, M. (2019). How do Swedish Government agencies define risk? *Journal of Risk Research*, 22(6), 717–734.

Bottou, L. (2012). Stochastic Gradient Descent Tricks. In *Neural Networks: Tricks of the Trade*, 7700, 421–436.

Charoenwong, C., Ding, D. K. and Jenwittayaroje, N. (2010). Price movers on the stock exchange of Thailand: Evidence from a fully automated order-driven market. *Financial Review*, 45(3), 761–783.

Chawda, B. V. and Madhubhai P. J. (2015). Stock market portfolio management: A walk-through. *International Journal on Recent and Innovation Trade in Computing and Communication*, 3(6), 4136-4143.

Chen, S. D. and Lim, A. E. B. (2020). A Generalized Black-Litterman Model. *Operations Research*, 68(2), 381–410.

Cheung, W. (2010). The Black-Litterman model explained. *Journal of Asset Management*, 11, 229-243.

Chin, L., Chendra, E. and Sukmana, A. (2018). Analysis of portfolio optimization with lot of stocks amount constraint: Case study index LQ45. *4th International Conference Operational Research*, 300, 012004.

Choi, T. and Chow, P. (2008). Mean-variance analysis of quick response program. *International Journal of Production Economics*, 114(2), 456-475.

Chollet, F. (2021). *Deep Learning with Python*. 2nd ed. USA: Simon and Schuster.

Choudhry, M. (2006). *An introduction to Value-at-Risk.* 4th ed. UK: John Wiley & Sons.

Christodoulakis, G. A. (2002). Bayesian Optimal Portfolio Selection: the Black-Litterman Approach. Unpublished paper.

Christoffersen, P. F. (2012). *Elements of Financial Risk Management*. USA: Academic Press.

Cong, F. and Oosterlee, C. W. (2016). Multi-period mean-variance portfolio optimization based on Monte-Carlo simulation. *Journal of Economic Dynamics and Control*, 64, 23-38.

Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, 1(2), 223-236.

Cornuejols, G. and Tutuncu, R. (2006). Optimization Methods in Finance. 1st ed. England: Cambridge Unibersity Press.

Dong, Y. and Peng, C. Y. J. (2013). Principled missing data methods for researchers. *SpringerPlus*, 2, 222.

Dorion, C. and Bengio, Y. (2003). *Stochastic Gradient Descent on a Portfolio Management Training Criterion Using the IPA Gradient Estimator*. CIRANO: Working Papers 2003s-23.

Duffie, D. and Pan, J. (1997). An overview of value of risk. *Journal of Derivatives*, 4(3), 7-49.

Durall, R. (2022). Asset Allocation: From Markowitz to Deep Reinforcement Learning. Retrieved on July 1, 2020, from https//doi.org/10.48550/arXiv.2208.07158.

Elton, E. J. and Gruber, M. J. (1997). Modern portfolio theory, 1950 to date. *Journal of Banking and Finance*, 21(11-12), 1743–1759.

Elton, E. J., Gruber, M. J., Brown, S. J. and Goetzmann, W. N. (2014). *Modern Portfolio Theiry and Investment Anaylsis*. 9th ed. United States: John Wiley & Sons.

Erdas, M. L. (2020). Developing a portfolio optimization model based on linear programming under certain constraint: An application on Borsa Istanbul 30 index. *Turkish Journal of TESAM Academy*, 7(1), 115–141.

Fabozzi, F. J., Kolm, P. N., Pachamanova, D. A. and Focardi, S. M. (2007). *Robust Portfolio Optimization and Management*. John Wiley & Sons.

Fadadu, P., Mathukiya, H. and Parmar, C. (2015). Portfolio selection: Using Markowitz model on selected sectors companies in India. *International Multidisciplinary Research Journal*, 2(12), 1-6.

Fahmy, H. (2020). Mean-variance-time: An extension of Markowitz's mean-variance portfolio theory. *Journal of Economics and Business*, 109, 105888.

Farias, C. A., Vieira, W. C. and Santos, M. L. (2006). Portfolio selection models: comparative analysis and applications to the Brazilian stock market. *Brazilian Review of Economics and Agribusiness*, 4(3), 387–407.

Fogarasi, J., Domán, C., Lámfalusi, I. and Kemény, G. (2015). Financial risk in Hungarian agro-food economy. *Management International Conference*, 453-459.

Ge, R., Huang, F., Jin, C. and Yuan, Y. (2015). Escaping From Saddle Points — Online Stochastic Gradient for Tensor Decomposition. Retrieved on July 1, 2020, from http://arxiv.org/abs/1503.02101.

Geert Rouwenhorst, K. (1999). European equity markets and the EMU. *Financial Analysts Journal*, 55(3), 57–64.

Gencay, R. and Selcuk, F. (2004). Extreme value theory and value-at-risk: Relative performance in emerging markets. *International Journal of Forecasting*, 20, 287-303.

Gilli, M., Maringer, D. and Schumann, E. (2019). *Numerical Methods and Optimization in Finance*. 2nd ed. USA: Academic Press.

Gough, J. D. (1988). *Risk and Uncertainty*. Lincoln University: Centre for Resource Management Information Paper Series.

Guo, W., Wang, Y. and Qiu, D. (2020). Mean-variance portfolio choice with uncertain variance-covariance matrix. *Journal of Financial Risk Management*, 9(2), 57-81.

Gupta, D., Sharma, M. and Trivedi, A. S. (2016). Risk management: Identifying key risks in construction projects. *International Journal of Civil and Structural Engineering Research*, 4(1), 9-15.

Gupta, K. and Chatterjee, N. (2019). *Stocks Recommendation Strategy Based on a Comparison between Large Number of Stocks*. Retrieved on January 25, 2022, from https://arxiv.org/ftp/arxiv/papers/1901/1901.11013.pdf

Gupta, P., Mehlawat, M. K., Yadav, S. and Kumar, A. (2020). Intuitionistic fuzzy optimistic and pessimistic multi-period portfolio optimization models. *Soft Computing*, 24(16), 11931-11956.

Hall, M. and Weiss, L. (1967). Firm size and profitability. *The Review of Economics and Statistics*, 49(3), 319–331.

Hassan, S. and Othman, Z. (2018). Forecasting on the long-term sustainability of the employees provident fund in Malaysia via the Box-Jenkins' ARIMA model. *Business and Economic Horizons*, 14, 43-53.

Haber, J. and Braunstein, A. (2009). Examining the role of short-term correlation in portfolio diversifiction. *Graziadio Business Review*, 12(3). Retrieved on July 1, 2020, from https://gbr.pepperdine.edu/2010/08/correlated-assets/

He, G. and Litterman, R. (1999). The Intuition Behind Black-Litterman Model Portfolios. Retrieved on July 1, 2020, from http://dx.doi.org/10.2139/ssrn.334304.

Idzorek, T. (2019). A Step-by-step guide to the Black-Litterman Model Incorporating User-specified Confidence Levels. Retrieved on July 1, 2020, from http://dx.doi.org/10.2139/ssrn.3479867.

Ishibashi, N. (2017). *Understanding Adversarial Training: Improve Image Recognition Accuracy of Convolution Neural Network*. City University of New York : Master Thesis.

Islam, M. R. (2018). Sample size and its role in Central Limit Theorem (CLT). *International Journal of Physics and Mathematics*, 1(1), 37–47.

Iyengar, G. and Ma, A. K. C. (2013). Fast gradient descent method for mean-CVaR optimization. *Annals of Operations Research*, 205, 203-212.

Jaafar, R. and Daly, K.J. (2019). The sustainability of Malaysia's defined contribution pension system: implementation of deterministic linear programming. *International Journal of Innovative Technology and Exploring Engineering*, 8, 97-103.

Jacquier, E., Kane, A. and Marcus, A. J. (2019). Geometric or arithmetroc mean: A recondiferation, *Financial Analysis Journal*, 59(6), 46-53.

Jais, I. K. M., Ismail, A. R. and Nisa, S. Q. (2019). Adam optimization algorithm for wide and deep neural network. *Knowlege Engineering and Data Science*, 2(1), 41-46.

Jorion, P. (1996). Risk2: Measuring the risk in value at risk. *Financial Analysts Journal*, 52(6), 47-56.

Kabir, S. H., Bacha, O. I. and Masih, M. (2013). Are Islamic equities immune to global financial turmoil? An investigation of the time varying correlation and volatility of Islamic equity returns. *Australian Journal of Basic and Applied Sciences*, 7(7), 686-701.

Kaplanski, G. and Kroll, Y. (2002). VaR risk measures versus traditional risk measures: An analysis and survey. *Journal of Risk*, 4(3), 1-27.

Keskar, N. S. and Socher, R. (2017). Improving Generalization Performance by Switching from Adam to SGD. Retrieved on July 1, 2020, from http://arxiv.org/abs/1712.07628.

Khindanova, I., Rachev, S. and Schwartz, E. (2001). Stable modeling of value at risk. *Mathematical and Computer Modelling*, 34, 1223-1259.

Kiefer, J. and Wolfowitz, J. (1952). Stochastic estimation of the maximum of a regression function. *Annals of Mathematical Statistics*, 23(3), 462-466.

Kingma, D. P. and Ba, J. L. (2015). Adam: A Method for Stochastic Optimization. *The International Conference on Learning Representations*. Retrieved on July 1, 2020, from http://arxiv.org/abs/1412.6980.

Krishnan, H. P. and Mains, N. E. (2005). The two-factor B-L model. Risk, 69-73.

KWSP (2022). *About EPF*. Retrived on October 30, 2021, from https://www.kwsp.gov.my/about-epf/corporate-profile.

Lasher, W. R. (2014). *Practical Financial Management*. 8th Ed. Florence, U.S.: Cengage Learning, Inc.

Le, A. and Stenius, A. (2017). *Equity Valuation Using Discounted Cash Flow Method-A case study: Viking Line Ltd*. Arcada: Degree‘s Project Report.

Lee, D. K., In, J. and Lee, S. (2015). Standard deviation and standard error of the mean. *Korean Journal of Anesthesiology*, 68(3), 220-223.

Lian, X., Wang, M. and Liu, J. (2017). Finite-sum Composition Optimization via Variance Reduced Gradient Descent. *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics*, PMLR 54, 1159-1167.

Li, B. and Teo, K.L. (2021). Portfolio optimization in real financial markets with both uncertainty and randomness. *Applied Mathematical Modelling*, 100, 125-137.

Li, Y., Li, A. and Liu, Z. (2018). Two ways of calculating VaR in risk management - An empirical study based on CSI 300 Index. *Procedia Computer Science*, 139, 432-439.

Liu, M. Yao, D., Liu, Z., Guo, J. and Chen, J. (2023). An improved Adam optimization algorithm combining adaptive coefficients and composite gradients based on randomized block coordinate descent. *Computational Intelligence and Neuroscience*, 2023, 4765891.

Mariak, V. and Mitková, Ľ. (2016). Long-term sustainability of portfolio investments-gender perspective: An overview study. *ACRN Oxford Journal of Finance and Risk Perspectives*, 5(1), 219-226.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77-91.

Menshawy, A. (2018). *Deep Learning By Example: A hands-on guide to implementing advanced machine learning algorithms and neural networks*. U.K.: Packt Publishing Ltd.

Mohamad, S., Hassan, T. and Sori, Z. M. (2006). Diversification across economic sectors and implication on portfolio investments in Malaysia. *Journal of Economics and Management*, 1(1), 155-172.

Morgan, D. L. (1996). Focus groups. *Annual Review of Sociology*, 22, 129-152.

Muhamedyev, R. I. (2015). Machine learning methods: An overview. *Computer Modelling and New Technologies*, 19(6), 14-29.

Muteba Mwamba, J. and Sutene, M. (2010). *An Alternative to Portfolio Selection Problem Beyond Markowitz's: Log Optimal Growth Portfolio*. MPRA Paper, No. 50240.

Netrapalli, P. (2019). Stochadtic gradient descent and its variants in machine learning. *Journal of the Indian Institute of Science*, 99, 201-213.

Okoth, O. W. (2020). Application and critism of mean variance theory. *International Journal of Social Science and Humanities Research*, 8(1), 493-498.

Olsson, S. and Trollsten, V. (2018). *The Black Litterman Asset Allocation Model An Empirical Comparison of Approaches for Estimating the Subjective View Vector and Implications for Risk-Return Characteristics*. Linköping University: Master Thesis.

Omisore, I., Yusuf, M. and Christopher, N. (2012). The modern portfolio theory as an investment decision tool. *Journal of Accounting and Taxation*, 4(2), 19-28.

Östergård-Hansen, J., Pipinytė, A. and Landström, J. (2017). *The Art of Diversifying an Investment Portfolio with Art*. Uppsala University: Master Thesis.

Paramasivan, C. and Subramanian, T. (2009). *Financial Management*. New Delhi: New Age International Pubblishers.

Paul, P. (2015). *How to Avoid Loss and Earn Consistently in the Stock Market: An Eas-to-Understand and Practical Guide for Every Investor*. 3rd Ed. India: Partridge Pub.

Petros, J. (2011). An empirical investigation of Markowitz Modern Portfolio Theory: A case of the Zimbabwe stock exchange. *Journal of Case Research in Business and Economics*. 1-16.

Polyak, B. T. (1964). Some methods of speeding up the convergence of iteration methods. *USSR Computational Mathematics and Mathematical Physics,* 4(5), 1-17.

Pratt, S. and Grabowski, R. J. (2010). *Cost of Capital in Litigation: Applications and Examples*. 4th Ed. Hoboken: John Wiley & Sons.

Qi, J. (2011). *Risk Measurement with High-frequency Data-Value-at-Risk and Scaling Law Methods*. University of Essex: Ph.D. Thesis.

Rani, A. (2012). The modern portfolio theory as an investment decision tool. *International Journal of Management Research and Review*, 2(7), 1164-1172.

Rashid, M. and San, W. K. C. (2019). Employee provident funds' market performance: the case of Malaysia, In Ghazali,, E.M., Mutum, D.S., Rashid, M., Ahmed, J.U. (eds) *Management of Shariah Compliant Business*. Management for Professionals. Switzerland: Springer, Cham.

Robbins, H. and Monro, S. (1951). A stochastic approximation method. *Annals of Mathematical Statistics*, 22(3), 400-407.

Robiyanto, R., Nugroho, B. A., Handriani, E. and Huruta, A. D. (2020). Hedge effectiveness of put replication, gold, and oil on ASEAN-5 equities. *Financial Innovation*, 6, 53, 1-29.

Roset, L. M. (2019). *Applications of Machine Learning to Studies of Quantum Phase Transitions*. The Barcelona Institute of Science and Technology: Master Thesis.

Rubinstein, M. (2002). Markowitz's "portfolio selection": A fifty-year retrospective. *The Journal of Finance*, 57(3), 1041-1045.

Ruder, S. (2016). An overview of gradient descent optimization algorithms. Retrieved on July 1, 2020, from http://arxiv.org/abs/1609.04747.

Rugman, A. M. (1976). Risk reduction by international diversification. *Journal of International Business Studies*, 7(2), 75-80.

Rutkauskas, A. V. and Stankeviien, J. (2003). Formation of an investment portfolio adequate for stochasticity of profit possibilities. *Journal of Business Economics and Management*, 4(1), 3-12.

Ryu, E. K. and Boyd, S. (2017). Stochastic Proximal Iteration: A Non-Asymptotic Improvement Upon Stochastic Gradient Descent. Retrieved on July 1, 2020, from https://web.stanford.edu/~boyd/papers/pdf/spi.pdf

Santiago, R. D. and Estrada, J. (2013). Geometric mean maximization: Expectation, observed, and simulated performance. *The Journal of Investing Summer*, 22(2), 109-119.

Satchell, S. and Scowcroft, A. (2000). A demystification of the Black–Litterman model: Managing quantitative and traditional portfolio construction. *Journal of Asset Management*, 1(2), 138-150.

Senthilnathan, S. (2015). Risk, return and portfolio theory - A contextual note. *International Journal of Science and Research*, 5(10), 705-715.

Solnik, B. H. (1974). Why not diversify internationally rather than domestically? *Financial Analysts Journal*, 30(4), 48-54.

Subekti, R., Ratna Sari, E., and Kusumawati, R. (2019). Combining Black-Litterman model with clustering on portfolio construction. *Journal of Physics: Conference Series*, 1321(2), 022051.

Sun, R. Y. (2020). Optimization for deep learning: An overview. *Journal of the Operations Research Society of China*, 8(2), 249-294.

Tölgyesi, C. and Pénzes, Z. 2018. (2018). *Biostatistics*. University of Szeged.

Vasigh, B. and Gorjidooz, J. (2016). *Engineering Economics for Aviation and Aerospace*. 1st ed. London: Routledge.

Van, B. J. Broeders, D., De Jong, M. and Koijen, R. (2014). Collective pension schemes and individual choice. *Journal of Pension Economics and Finance*, 13(2), 210-225.

Van der Geugten, G. (2015). *Online Learning Algorithms: Methods and Applications*. Delft University of Technology: Bachelor Thesis.

Yu, P., Lee, J. S., Kulyatin, I., Shi, Z. and Dasgupta, S. (2019). Model-based Deep Reinforcement Learning for Dynamic Portfolio Optimization. Retrieved on July 1, 2020, from http://arxiv.org/abs/1901.08740.

Zhang, J., Leung, T. and Aravkin, A. (2020). Sparse mean-reverting portfolios via penalized likelihood optimization. *Automatica*, 111, 108651.

Zhang, Z. (2011). *Dynamic Coherent Acceptability Indices and Their Applications in Finance*. Illinois Institute of Technology: Ph.D. Thesis.

Zhou, G. (2009). Beyond Black-Litterman: Letting the data speak. *The Journal of Portfolio Management*, 36(1), 36-45.

Zhou, S., Shi, B. and Wen, Z. (2012). Analysis of mean-VaR model for financial risk control. *Systems Engineering Procedia*, 4, 40-45.

VITA

The author was born on April 12, 1992, in Sarawak, Malaysia. She went to Ranan Junior High School in Sibu, Sarawak, Malaysia. In 2012, she pursued the Bachelor of Science (Industrial Statistics) with Honours at Universiti Tun Hussein Onn Malaysia and graduated in 2016. She then enrolled at the same university to study for the Doctor of Philosophy (PhD) in Science in 2016. During her study, she participated in the 7th International Conference on Optimization and Control with Application (OCA) as a presenter in 2018. In the following year, she published her first paper in a scientific journal entitled "Using Historical Return Data in the Black- Litterman Model for Optimal Portfolio Decision". Fortunately, this paper was awarded Best Article Journal on Science Technology & Medicine (National Index) in the MAPIM-KPT Awards 2019. She published her second journal paper "An Improvement of Stochastic Gradient Descent Approach for Mean-Variance Portfolio Optimization Problem" in the Journal of Mathematics in 2019. In the same year, she participated in the 27th National Symposium on Mathematical Sciences.