DEVELOPING ALGORITHMS AND COMPARING FILTERING TECHNIQUES FOR THREE STOCHASTIC OPTIMAL CONTROL PROBLEMS IN FINANCE

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Dedicated to my most respected supervisor, Dr. Kek Sie Long,
for being my guardian who led me throughout the research study.

To my beloved parents,
who gave birth of me and offer their unconditional love,
respect as well as support to all decisions I made.

Along with my friends,
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ABSTRACT

Optimization and control of a nonlinear dynamical system that is disturbed by random noises is a challenging task. In this thesis, the application of Kalman filtering techniques is aimed at solving stochastic optimal control problems in economics and finance. For this purpose, the extended Kalman filter for state-control (EKFSC) and unscented Kalman filter for state-control (UKFSC) algorithms are developed to associate state estimation and optimal control law. In the EKFSC algorithm, the state equation is propagated and linearized. Then, the updates on output measurement and time are taken to estimate the state dynamics. While, in the UKFSC algorithm, the unscented transform is applied to the state equation so that a set of sigma points is generated. Based on these sigma points, the state dynamics are predicted through updating output measurement and time. By applying the state estimates, the optimal control law is designed properly. Here, the state estimate and the optimal control are assumed to follow the principle of separation. For illustration, three stochastic optimal control problems, which are economic growth, financial risk, and chaotic and hyperchaotic financial models, are studied. The simulation results showed that the state dynamics of these models are well-estimated by the UKFSC algorithm with a smaller mean square error than the EKFSC algorithm. Furthermore, these dynamic systems using the UKFSC algorithm achieve better stability, and better optimal solutions than the EKFSC algorithm. In conclusion, the efficiency of the UKFSC algorithm for handling nonlinear stochastic optimal control problems in economics and finance is verified appropriately.

ABSTRAK

Pengoptimuman dan pengawalan sistem dinamik tak linear yang terganggu oleh hingar rawak merupakan tugas yang mencabar. Dalam tesis ini, penggunaan teknik penapisan Kalman bertujuan untuk menyelesaikan masalah kawalan optimum stokastik dalam ekonomi dan kewangan. Untuk tujuan ini, algoritma penapis Kalman lanjutan bagi keadaan-kawalan (EKFSC) dan penapis Kalman tanpa wangian bagi keadaan-kawalan (UKFSC) telah dibangunkan untuk menggabungkan anggaran keadaan dan kawalan optimum. Dalam algoritma EKFSC, persamaan keadaan dirambatkan dan terlinear. Kemudian, pengemaskinian pada pengukuran keluaran dan masa dilakukan untuk menganggarkan dinamik keadaan. Manakala, dalam algoritma UKFSC, penjelmaan tanpa wangian digunakan pada persamaan keadaan supaya satu set titik sigma dapat dijanakan. Berdasarkan titik sigma ini, dinamik keadaan diramalkan melalui pengemaskinian pengukuran keluaran dan masa. Dengan menggunakan anggaran keadaan, hukum kawalan optimum direkabentuk dengan sewajarnya. Dengan ini, anggaran keadaan dan kawalan optimum diandaikan mengikuti prinsip pemisahan. Sebagai penerangan, tiga masalah kawalan optimum stokastik, iaitu pertumbuhan ekonomi, risiko kewangan dan model kewangan berkalut and hiperkalut, telah dikaji. Keputusan simulasi menunjukkan bahawa dinamik keadaan model-model tersebut dianggarkan dengan baik oleh algoritma UKFSC dengan ralat min kuasa dua yang lebih kecil daripada algoritma EKFSC. Tambahan pula, sistem dinamik tersebut yang menggunakan algoritma UKFSC dapat mencapai kestabilan dan penyelesaian optimum yang lebih baik daripada algoritma EKFSC. Kesimpulannya, kecekapan algoritma UKFSC untuk mengendalikan masalah kawalan optimum stokastik tak linear dalam ekonomi dan kewangan disahkan dengan jelas.

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LIST OF SYMBOLS

α	-	step-size
i	-	iteration number
f	-	plant function
h	-	output measurement channel
ω	-	Gaussian white noises
Q	-	covariance matrices
φ	-	terminal cost
L	-	cost under summation
J	-	total cost to be minimized
E	-	expectation operator
u	-	control sequence
x	-	state sequence
y	-	output sequence
M	-	error covariance matrix
$\bar{x}$	-	state mean sequences (predicted state)
J_{lse}	-	weighted least-squares error
$\hat{x}$	-	optimal state estimate (filtered state/ updated mean)
$\bar{y}$	-	output estimate (expected output sequence)
K	-	Kalman filter gain
P	-	predicted covariance
M_x	-	predicted state error covariance
M_y	-	output error covariance
χ	-	sigma points
λ	-	scaling factor
κ	-	secondary scaling factor

n	-	dimension of the state vector
W	-	weight value of sigma points
Υ	-	set of transformed sigma points
R	-	output noise covariance
$\hat{y}$	-	predicted observation
P_{yy}	-	observation error covariance
P_{xy}	-	cross-correlation matrix
H	-	Hamiltonian function
p	-	costate sequence
J'	-	augmented cost function
A	-	state transition matrix
B	-	control coefficient matrix
S	-	Riccati solution
g	-	gradient of the cost function
$\Re$	-	set of real numbers
arg min	-	minimum value of an argument
P_x	-	state error covariance
P_y	-	output error covariance
T	-	transposition operator

LIST OF ABBREVIATIONS

ARCH	-	autoregressive conditional heteroscedastic
EKF	-	extended Kalman filter
EKFSC	-	extended Kalman filter for stochastic control
KF	-	Kalman filter
LQE	-	linear quadratic estimation
LQG	-	linear quadratic Gaussian
LQR	-	linear quadratic regulator
MSE	-	mean square error
ODE	-	ordinary differential equation
PDE	-	partial differential equation
R&D	-	research and development
SDE	-	stochastic differential equations
SSE	-	sum of square error
UKF	-	unscented Kalman filter
UKFSC	-	unscented Kalman filter for stochastic control
UT	-	unscented transformation

LIST OF PUBLICATION

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CHAPTER 1

INTRODUCTION

This chapter provides a general introduction to the study in this thesis. Also, the background of the study is delivered, and the problem statement is presented. Then, the objectives of the study are established, and the scope of the study is mentioned. Later, the significance of the study is highlighted and the content of each chapter in this thesis is outlined.

1.1 General introduction

Over the past few decades, optimal control has become a widely developed research frontier with many contributions to the theory and applications in both deterministic and stochastic cases (Fleming and Rishel, 2012). Meanwhile, optimal control models play a prominent role in many application areas, including aerospace, robotics, engineering, and sciences (Kafash and Nadizadeh, 2017). Moreover, the optimal control theory has been applied in business, finance, and economics (Craven and Islam, 2002). Numerous optimal control problems in the real world are considerably more complex when involving a dynamic system that has not been well-defined to obtain an analytical solution. Hence, numerical algorithms have become a popular approach to solving optimal control problems.

In fact, an optimal control problem requires the identification of a practical approach to achieve the optimal possible outcome of a dynamic system. More formally, an optimal control problem means that a parameter in a mathematical model is endogenously controlled to achieve an optimal output using optimization techniques (Torre *et al.*, 2015). Basically, the optimal control theory is a branch of applied mathematics and engineering that deals with finding a feedback control law for a dynamical system over time in which to minimize a cost function (Naidu, 2002). From the evolution point of view, the optimal control is an extension of the calculus of variations and is a mathematical dynamic optimization method for deriving control policies (Sargent, 2000).

In optimal control, a dynamical system, which is a class of differential equations, is under consideration such that a set of the certain optimality conditions is satisfied. In addition, a cost function in terms of state and control variables would be minimized. The state variable is the set of variables that is used to describe the mathematical state of a system, while the control variable is an operation policy that regulates and monitors the processing and the transmission of data (Rose, 2015). With the existence of the admissible control law, the stability of the dynamical system can be ensured. Thus, the minimum cost function is the best performance index to measure the efficiency of the dynamical system in an optimal control problem.

Moreover, optimization and control of a dynamical system, which is disturbed by random noises, are very challenging. This is because in presence of the random effect of noises, the fluctuation behaviour that arises, either naturally or intentionally, would lead the dynamical system to an undesired solution. This issue has quite commonly happened in economics and finance. This problem is formally known as a stochastic optimal control problem. Hence, the formulation of a mathematical model for studying such a system is really crucial and required in order to contribute some significant results to the communities of economics and finance.

1.2 Background of study

Control theory is a mathematical description of acting optimally in obtaining the future gain. The optimal control theory is an extension of the calculus of variations that has a fruitful history stretching back over 380 years with contributions from Isaac Newton and Johann Bernoulli in the 17^{th} century. Euler and Lagrange developed the calculus of variations in the 18^{th} century, then it was contributed by Andrien Legendre, Carl Jacobi, William Hamilton, and Karl Weierstrass in the 19^{th} century. The further expansion of optimal control includes the formulation of dynamic programming by Richard Bellman in the 1950s, linear quadratic regulator (LQR) and the Kalman filter (KF) by Rudolf Kalman in the 1960s (Ho and Kalman, 1966).

There are several applications of control theory in fields such as economy, mathematical finance, and engineering. Historically, the classical theory of control for deterministic, linear, time-invariant systems was established during the 1930s and 1940s. Later, this theory was extended to linear, time-invariant systems involving stationary random signals using prediction and filtering theory during the 1940s and 1950s. Afterwards, the fundamental trend of research in control theory has been revolved toward the modern theory of optimal control. With the application of state-space techniques, calculus of variations, dynamic programming and a comprehensive theory for deterministic systems exists. The extension of the modern optimal control theory is a potentially fruitful area of research as the classical theory has been extended to include the effects of random disturbances. Hence, optimal control theory has recently sparked a growing interest in the mathematics and control communities.

In mathematical definition, optimal control is the process of determining control and state trajectories for a dynamical system over time to minimize a performance index. The formulation of an optimal control problem requires a mathematical model of the system to be controlled, performance index, boundary conditions on state, and constraints to be satisfied by the states and control variables. On this basis, deterministic and stochastic control systems are two main areas of research. From here, optimal control is evolved rapidly in the development of theories, algorithms, and applications.

Stochastic optimal control is one of the active research areas in control theory. Unlike the deterministic control, stochastic optimal control deals with the existence of uncertainty in observation and random disturbance in a dynamical system (Ahmed, 1973). The problem of stochastic optimal control is described as determining a set of admissible control to minimize a cost function in its expectation form over a general class of differential or difference equations in presence of the random disturbances (Adomian, 1985). The existence of uncertainty, either in observation or in random disturbance, drives the evolution of a system disturbed by random noises. Here, the random noise with a known probability distribution is assumed to affect the evolution and observation of the state variables in the dynamical system.

Since the stochastic optimal control aims to design the time path of the control variables, either discrete or continuous time, so the desired control function can be performed by considering the random disturbances with minimum cost regardless of the existence of the random noise. This could be done when the nonlinear filtering theory is employed to estimate the state dynamics and then the control policy is designed based on the state estimates. However, using the nonlinear filtering theory involves a high computational cost.

Therefore, applying the Kalman filtering theory to stochastic optimal control with Gaussian white noises greatly benefits the state estimation and optimal control design. The calculation procedure for state estimation has been simplified and becomes tractable, where the linearization of state equation is considered. However, the issue of less accuracy in state estimates, which is resulted from the linearization, raises the motivation to study a better state estimation strategy for the control policy design in solving the stochastic optimal control problems.

1.3 Problem statement

In previous studies, many computational approaches have been developed for solving stochastic optimal control problems, both for linear and nonlinear cases. However, the state dynamics that are disturbed by random noises show the behaviour of fluctuation and would not be estimated appropriately. By virtue of this, the exact optimal solution to stochastic optimal control problem is impossible to be obtained. Moreover, the state dynamics that are estimated using the nonlinear filtering theory would be computationally demanding and the design of optimal control law is less accurate in practical applications. Hence, in handling these weaknesses, the efficient computational algorithms are necessarily required.

Indeed, under the separation principle, the state dynamics, which is normally estimated by using the Kalman filter theory, and the feedback optimal control law, which is designed upon state estimation, are assumed to be determined separately. The application of the Kalman filtering theory in state estimation for a linear dynamical system with Gaussian white noise is quite satisfactory, where the optimal state estimate could be obtained. Unfortunately, the result of state estimation for nonlinear dynamical systems using the extended Kalman filter (EKF) provides a sub-optimal state estimate through approximation of the first-order linearization or even fails to give any state estimate. From the problem described above, it is noticed that optimizing and controlling nonlinear stochastic dynamical system shall be further investigated by proposing an efficient state estimation approach.

Hence, the main aim of the study in this thesis is to apply the extended Kalman filter (EKF) and the unscented Kalman filter (UKF) techniques for solving nonlinear stochastic optimal control problems in finance. By devoting to this aim, two computational algorithms, which are the EKF for state-control (EKFSC) algorithm and the UKF for state-control (UKFSC) algorithm, are developed. The specificity of both algorithms is to associate state estimation and optimal control in handling stochastic optimal control problems. Also, Gaussian random disturbances in the real financial market reflect uncertain dynamic nature of financial problems that are difficult to predict efficiently using general statistical-based and mathematical-based models. Therefore, this study applies these two filtering techniques in estimating the state and controlling the dynamic systems. Not only this, three recent top concern financial problems, which are economic growth, financial risk, and financial chaotic models, are chosen as these issues affect the growth of developing countries.

1.4 Objectives of study

The objectives of the study in this thesis are established as follows:

(a) To develop computational algorithms, which are the EKFSC and UKFSC algorithms, for state estimation and optimal control law in solving nonlinear stochastic optimal control problems.

(b) To compare the accuracy and efficiency of the proposed filtering techniques in optimizing and controlling three stochastic optimal control problems in finance, which are economic growth, financial risk, and financial chaos models.

1.5 Scope of study

This study explores the application of the computational algorithms, which are the EKFSC and UKFSC algorithms, for solving nonlinear stochastic optimal control problems in three financial models, which are economic growth, financial risk, and financial chaotic systems. At first, this study covers a brief review of literature for understanding stochastic optimal control problems. Then, this study provides the methodology of the EKFSC and UKFSC algorithms in estimating the state dynamics and designing the control law for solving stochastic optimal control problems.

Considering the presence of Gaussian random noise during the problem formulation, the Kalman filtering techniques, which are the EKF and the UKF, are the appropriate filtering techniques that can be used in this study.

Further, the proposed computational techniques are applied to illustrate the financial applications in economic growth, financial risk, chaotic and hyperchaotic financial systems. These financial systems are studied as economic growth is important to determine the wealth of a country, whereas financial risk states the potential loss of financial investment, and financial chaotic systems display the unpredictable and complex financial systems. Since these financial problems are the top three concern issues in the community of economics and finance, the simulation results obtained in this study can provide some meaningful insights from the mathematical point of view.

1.6 Significance of study

In this study, two efficient computational algorithms for state estimation, which satisfies the certainty equivalence property, are proposed to solve the stochastic optimal control problems. The methodology of these algorithms is discussed and the application of these algorithms in finance is verified. Thus, some contributions of the study are highlighted as follows,

(a) The efficient computational algorithms, which are the EKFSC algorithm and the UKFSC algorithm are proposed for solving nonlinear stochastic optimal control problems. The state estimation and the optimal control law are associated in each algorithm to solve nonlinear stochastic optimal control problems.

(b) The improvement on state estimation for designing feedback optimal control law is carried out. This improvement is addressed in the UKFSC algorithm, which is applying the unscented transformation for state estimation rather than using the linearization as applied in the EKFSC algorithm.

(c) Illustrative examples of stochastic optimal control problems in the financial application are studied. These examples, which are economic growth, financial risk, chaotic and hyperchaotic financial models, are discussed. With these illustrative examples, the efficiency of the algorithms proposed is proven.

Hence, the outcome of the study is used to provide an optimal decision for stochastic optimal control problems, where the decision is very useful in many disciplines, particularly in applications of economics and finance.

1.7 Outline of thesis

This thesis consists of seven chapters. In Chapter 1, a general introduction to optimal control for both deterministic and stochastic cases is delivered. The background of the study, which expresses the study to be carried out, is mentioned. The problem statement in dealing with nonlinear stochastic optimal control problems is clarified. Next, the objectives of the study, which are the purposes for resolving the problem mentioned, are established. In addition, the scope of the study is provided, and the significance of the study is highlighted. Finally, the content of each chapter in this thesis is outlined and a summary is given.

In Chapter 2, the basic knowledge of stochastic dynamical systems is given to understand these systems easily. Then, the Kalman filtering (KF) techniques, which are applied for navigation, prediction, and filtering, are presented. In particular, the standard Kalman filter (KF), extended Kalman filter (EKF) and unscented Kalman filter (UKF) are mainly discussed. On the other hand, stochastic optimal control problems, which focus on economic growth, financial risk, chaotic and hyperchaotic financial systems, are reviewed.

In Chapter 3, the problem formulation reveals the general mathematical model in stochastic optimal control problems. The EKF and UKF techniques are discussed and explained. The calculation procedures are then summarized as the EKFSC algorithm and the UKFSC algorithm so that state estimation and optimal control are combined to handle nonlinear stochastic optimal control problems.

In Chapter 4, an economic growth system is studied. By considering the presence of random disturbances in the system, the stochastic optimal control problem for economic growth system is described. The simulation results are obtained by using the algorithms that are proposed in Chapter 3 and the graphical solutions are discussed for decision making.

In Chapter 5, a financial risk system is considered. The random noises are added to the system, and cause the system becomes more complex to be solved. So, the stochastic optimal control problem for the system is introduced aiming at state estimation and optimal control law design. The simulation results are determined by using the algorithms that have been discussed in Chapter 3. Hence, the optimal decision on this problem is suggested.

In Chapter 6, chaotic and hyperchaotic financial systems are concerned. With certain parameter values, these systems show chaotic behaviour, which lead the systems to be more complicated and unpredicted. By virtue of this, these systems are formulated as stochastic optimal control problems. Then, the algorithms that have been discussed in Chapter 3 are applied aiming at solving these problems. The simulation results are provided, and from the graphical solutions, the optimal decisions for these problems are given.

In Chapter 7, a conclusion of the study is given. The contributions of study, which are the efficient computational approach for state estimation, the optimal solution for nonlinear stochastic optimal control problems in financial applications, and the practicality of the proposed approaches for handling financial problems, are highlighted. The limitation of study is mentioned and some recommendations for future research are pointed out.

1.8 Summary

In this chapter, a general introduction of the study was given, and the background of the study was provided. The problem statement was described, and two objectives of the study were established. The scope of the study was mentioned, and the significance of the study was highlighted. The content of each chapter in the thesis was outlined. In the next chapter, the previous studies in the literature are reviewed.

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